

$\pi - \alpha$

$$\text{sen} (\pi - \alpha) = \text{sen } \alpha$$

$\pi + \alpha$

$$\text{sen}(\pi + \alpha) = -\text{sen } \alpha$$

$$\cos (-\alpha) = \cos \alpha$$

$$\operatorname{tg}(-\alpha) = -\operatorname{tg} \alpha$$

$$\frac{\pi}{2} - \alpha$$

$$\frac{\pi}{2} + \alpha$$

$$\sin\left(\frac{\pi}{2} + \alpha\right) = \cos \alpha$$

$$\operatorname{sen}\left(\frac{3\pi}{2}-\alpha\right)=-\cos \alpha$$

$$\cos\left(\frac{3\pi}{2}-\alpha\right)=-\operatorname{sen} \alpha$$

$$\operatorname{tg}\left(\frac{3\pi}{2}-\alpha\right)=\frac{1}{\operatorname{tg}\alpha}$$

$$\text{sen}\left(\frac{3\pi}{2} + \alpha\right) = -\cos \alpha$$

$$\cos\left(\frac{3\pi}{2} + \alpha\right) = \text{sen } \alpha$$

$$\operatorname{tg}\left(\frac{3\pi}{2} + \alpha\right) = -\frac{1}{\operatorname{tg} \alpha}$$

$$\operatorname{sen}(2\pi - \alpha) = -\operatorname{sen} \alpha$$

$$\cos(2\pi - \alpha) = \cos \alpha$$

$$\operatorname{tg}(2\pi - \alpha) = -\operatorname{tg} \alpha$$

$$\cos (180^\circ - \alpha) = -\cos \alpha$$

$$\text{sen } (180^\circ - \alpha) = \text{sen } \alpha$$

$$\operatorname{tg}(180^\circ - \alpha) = -\operatorname{tg} \alpha$$

$$\cos (180^\circ + \alpha) = -\cos \alpha$$

$$\text{sen } (180^\circ + \alpha) = -\text{sen } \alpha$$

$$\operatorname{tg}(180^\circ + \alpha) = \operatorname{tg} \alpha$$

$$\cos (-\alpha) = \cos \alpha$$

$$\text{sen } (-\alpha) = -\text{sen } \alpha$$

$$\operatorname{tg}(-\alpha) = -\operatorname{tg} \alpha$$

$$\cos (90^{\circ} - \alpha) = \operatorname{sen} \alpha$$

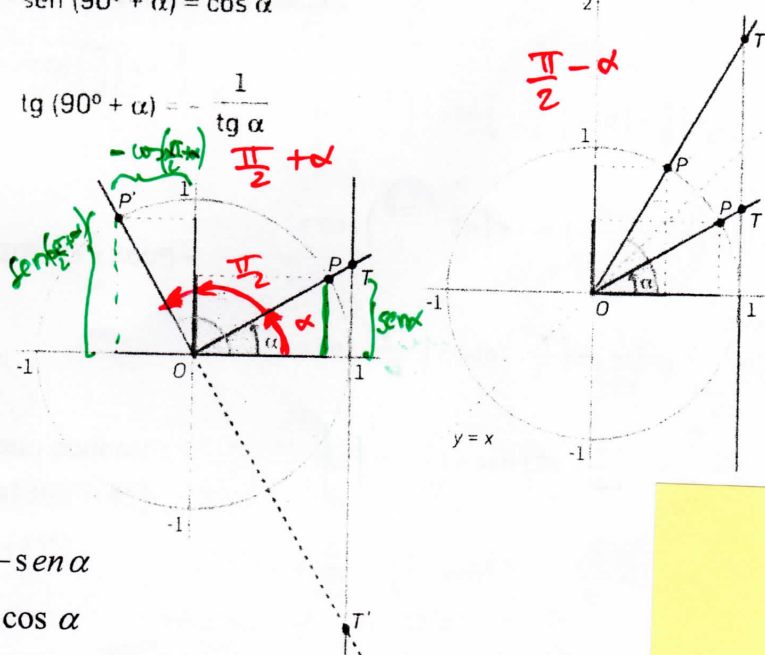
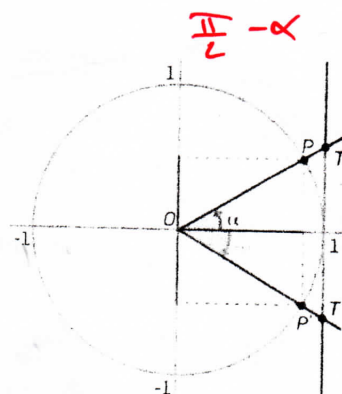
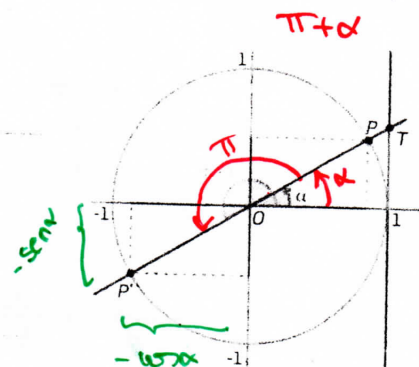
$$\text{sen } (90^\circ - \alpha) = \cos \alpha$$

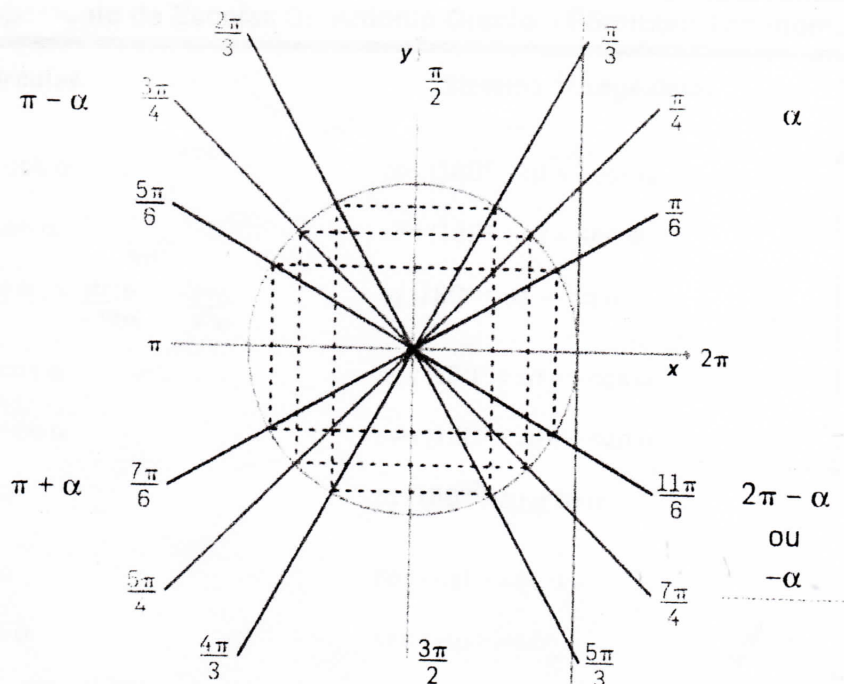
$$\operatorname{tg}(90^\circ - \alpha) = \frac{1}{\operatorname{tg} \alpha}$$

$$\cos (90^\circ + \alpha) = -\operatorname{sen} \alpha$$

$$\text{sen } (90^\circ + \alpha) = \cos \alpha$$

$$\operatorname{tg}(90^\circ + \alpha) = -\frac{1}{\operatorname{tg} \alpha}$$





Como determinar, sem recurso a calculadora, o valor de $\cos\left(\frac{16\pi}{3}\right)$? Vamos dar indicações gerais e aplicá-las a este caso particular.

- 1.º Se necessário, reduzir a amplitude do ângulo ao intervalo $[0, 2\pi[$, subtraindo ou adicionando múltiplos de 2π .

$$\cos\left(\frac{16\pi}{3}\right) = \cos\left(\frac{16\pi}{3} - 4\pi\right) = \cos\left(\frac{4\pi}{3}\right)$$

- 2.º Relacionar a amplitude obtida com uma amplitude α do 1.º quadrante, recorrendo às expressões: $\pi - \alpha$, $\pi + \alpha$ ou $-\alpha$.

$$\frac{4\pi}{3} = \pi + \frac{\pi}{3}$$

- 3.º Recorrendo ao círculo trigonométrico, aplicar as relações entre as razões trigonométricas de α e de $\pi - \alpha$, $\pi + \alpha$ ou $-\alpha$ e finalizar o cálculo.

$$\cos\left(\pi + \frac{\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$$

Exemplos

$$1. \text{ sen}(150^\circ) + \cos(315^\circ) - \text{tg}(1320^\circ) =$$

$$= \text{sen}(180^\circ - 30^\circ) + \cos(315^\circ - 360^\circ) - \text{tg}(1320^\circ - 3 \times 360^\circ) =$$

$$= \text{sen}(30^\circ) + \cos(-45^\circ) - \text{tg}(240^\circ) =$$

$$= \text{sen}(30^\circ) + \cos(45^\circ) - \text{tg}(180^\circ + 60^\circ) =$$

$$= \frac{1}{2} + \frac{\sqrt{2}}{2} - \text{tg}(60^\circ) =$$

$$= \frac{1}{2} + \frac{\sqrt{2}}{2} - \sqrt{3}$$

também poder ser
+ $\cos(360^\circ - 45^\circ)$
+ $\cos(45^\circ)$

$$2. \cos\left(\frac{8\pi}{3}\right) + \text{sen}\left(-\frac{9\pi}{4}\right) + \text{tg}\left(-\frac{\pi}{6}\right) =$$

$$= \cos\left(\frac{8\pi}{3} - 2\pi\right) + \text{sen}\left(-\frac{9\pi}{4} + 4\pi\right) - \text{tg}\left(\frac{\pi}{6}\right) =$$

$$= \cos\left(\frac{2\pi}{3}\right) + \text{sen}\left(\frac{7\pi}{4}\right) - \left(\frac{\sqrt{3}}{3}\right) =$$

$$= \cos\left(\pi - \frac{\pi}{3}\right) + \text{sen}\left(2\pi - \frac{\pi}{4}\right) - \frac{\sqrt{3}}{3} =$$

$$= -\cos\left(\frac{\pi}{3}\right) - \text{sen}\left(\frac{\pi}{4}\right) - \frac{\sqrt{3}}{3} =$$

$$= -\frac{1}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{3}$$

